

# Impact Ionization Can Explain Carrier Multiplication in PbSe Quantum Dots

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## ABSTRACT

The efficiency of conventional solar cells is limited because the excess energy of absorbed photons converts to heat instead of producing electron–hole pairs. Recently, efficient carrier multiplication has been observed in semiconductor quantum dots. In this process, a single, high-energy photon generates multiple electron–hole pairs. Rather exotic mechanisms have been proposed to explain the efficiency of carrier multiplication in PbSe quantum dots. Using atomistic pseudopotential calculations, we show here that the more conventional impact ionization mechanism, whereby a photogenerated electron–hole pair decays into a biexciton in a process driven by Coulomb interactions between the carriers, can explain both the rate ( $\ll 1$  ps) and the energy threshold ( $\sim 2.2$  times the band gap) of carrier multiplication, without the need to invoke alternative mechanisms.

Solar cells are a source of clean, sustainable energy, but to make them competitive with fossil fuels their efficiency must be increased significantly.<sup>1</sup> A major drawback of conventional solar cells is that high-energy photons at the blue end of the solar spectrum are largely wasted because their excess energy  $\hbar\omega - E_g$  (where  $\omega$  is the photon frequency and  $E_g$  is the solar-cell band gap) is converted into heat by electron–phonon scattering processes, instead of being used to produce additional electron–hole pairs. This efficiency bottleneck could be overcome by direct carrier multiplication.<sup>2–10</sup> In this process, a *single* absorbed high-energy photon generates *two or more* electron–hole pairs, thereby potentially increasing the output electrical current of the solar cell. In *bulk solids*, direct carrier multiplication is rather inefficient because competing decay channels for the photogenerated electron–hole pairs, such as phonon-assisted electron/hole cooling, are extremely fast. For example, in crystalline Si solar cells a maximum carrier multiplication efficiency of 30% at  $\hbar\omega = 4.6$  eV was reported,<sup>11</sup> meaning that on average one photon generates 1.3 excitons. In *quantum dots*, dramatically higher carrier multiplication efficiencies have been observed recently.<sup>5–9</sup> For example, Schaller et al.<sup>8</sup> reported quantum efficiencies as high as 700% in PbSe colloidal nanocrystals. However, the mechanism of carrier multiplication in quantum dots is still unknown.

In bulk solids, carrier multiplication can occur through a scattering mechanism known as impact ionization (II).<sup>12,13</sup> In this process (Figure 1), a photon of energy  $\hbar\omega > 2E_g$  creates a highly excited electron–hole pair, which in turn decays into a biexciton through an energy-conserving transition driven by many-body Coulomb interactions between carriers. However, II was dismissed<sup>6,7</sup> as a possible explanation of carrier multiplication in PbSe quantum dots on the basis that it would be too slow to account for the observed

high carrier multiplication efficiency. Instead, rather exotic explanations, involving resonance between single-exciton and multi-exciton states,<sup>6</sup> or second-order coupling with virtual excitons,<sup>7</sup> were tentatively offered. Indeed, pseudopotential calculations<sup>3,4</sup> of the II rate in CdSe quantum dots at threshold ( $\hbar\omega \approx 2E_g$ ) predicted a II lifetime  $\tau_{II} = 76$  ps, which is too slow to explain efficient carrier multiplication. However, the II rate at higher energy ( $\hbar\omega > 2E_g$ ) was not calculated. Recent tight-binding calculations<sup>14</sup> have suggested that the II rate at a given exciton energy is similar in PbSe quantum dots and in bulk PbSe. In this Letter we show, via atomistic pseudopotential calculations of the electronic structure of PbSe quantum dots, that the II mechanism can indeed explain the observed carrier multiplication lifetime and energy threshold. This different interpretation could lead to different optimization strategies for high-efficiency solar cells.

The experimental observation of carrier multiplication in quantum dots<sup>5–9</sup> has raised several interesting questions:

(i) *Asymmetry between carrier multiplication and Auger recombination rates*: Carrier multiplication was shown<sup>6,7</sup> to have a much shorter lifetime than the nonradiative decay of a biexciton into a single-exciton (Auger recombination, AR). The II decay rate of an exciton state  $|X_i\rangle$ , of energy  $E_i$  is given, according to the Fermi golden rule, by

$$\frac{1}{\tau_i^{II}} = \frac{2\pi}{\hbar} \sum_f |\langle X_i | W | XX_f \rangle|^2 \delta(E_i - E_f) \quad (1)$$

where the sum runs over the biexciton states  $|XX_f\rangle$  of energy  $E_f$  and  $W$  is the screened Coulomb interaction.<sup>15</sup> Conversely, the AR decay rate of a biexciton is given by

$$\frac{1}{\tau_i^{AR}} = \frac{2\pi}{\hbar} \sum_f |\langle XX_i | W | X_f \rangle|^2 \delta(E_i - E_f). \quad (2)$$

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Thus, the II and AR processes are described by the same transition matrix elements  $\langle X|W|XX\rangle$

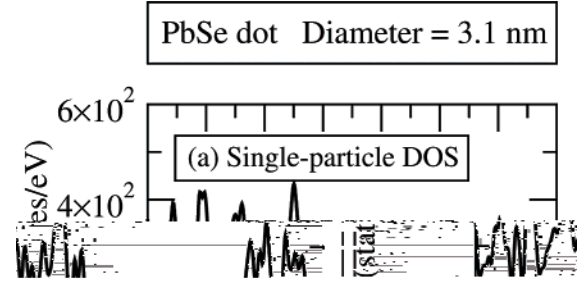
states (Figure 1e and f). Finally, the electron and the hole relax via electron/hole cooling, leading to a ground-state exciton (Figure 1g). In this Letter, we focus on the II and AR processes.

We consider a nearly spherical  $\text{Pb}_{260}\text{Se}_{249}$  quantum dot 3.1 nm in diameter. The dot is obtained by cutting out a Se-centered sphere from bulk PbSe (rocksalt lattice structure, lattice constant  $a_0 = 6.12 \text{ \AA}$ ). All Pb and Se surface atoms are passivated using ligand-like atomic pseudopotentials (centered along the direction of the dangling bonds), that are fitted to remove surface states from the dot band gap. The single-particle energies,  $\epsilon_i$ , and wave functions,  $\psi_i$ , are then obtained by solving the single-particle Schrödinger equation

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + \hat{V}_{\text{SO}} \right] \psi_i(\mathbf{r}) = \epsilon$$

where  $\hat{V}_{\text{SO}}$  is the nonlocal spin-orbit operator and  $V(\mathbf{r}) = \sum_{\mathbf{R}} v(\mathbf{r} - \mathbf{R})$  is a linear superposition of screened atomic pseudopotentials centered at the atomic positions  $\{\mathbf{R}\}$ . The Pb and Se atomic pseudopotentials are fitted to bulk PbSe transition energies and effective masses.<sup>17</sup> Equation 3 is solved by expanding the wave functions  $\psi_i(\mathbf{r})$  in a plane-wave basis set. We calculate all of the valence states and the conduction states in an energy window of  $\sim 5 \text{ eV}$  above the band gap, for a total of  $\sim 2000$  single-particle states. Our pseudopotential approach fully includes inter-valley coupling as well as inter-band coupling, both arising from the finite size of the quantum dot and the lack of translational symmetry.

Figure 2a shows the single-particle density of states (DOS) of the 3.1 nm quantum dot, calculated from eq 3. The band gap is  $E_g = 1.6 \text{ eV}$ . The most striking feature of the DOS is the very dense manifold of valence states, which form a quasi-continuum of levels near the valence-band edge. We find that the proposed mirrorlike symmetry between valence and conduction states, predicted by  $8 \times 8 \mathbf{k} \cdot \mathbf{p}$  calculations,<sup>16</sup> does not exist in a realistic atomistic pseudopotential calculation. This is a result of (i) the presence of valence states originating from multiple valleys in the Brillouin zone, located along the L-K and K-X lines, which are not included in  $\mathbf{k} \cdot \mathbf{p}$  calculations,<sup>16</sup> and (ii) the existence of a finite potential barrier at the surface of the dot, which leads to a more densely spaced manifold of valence-band energy levels compared to the case of an infinite potential barrier. Figure 2b shows densities of states of the mono-exciton (X) and the bi-exciton (XX), calculated as  $\rho_X(E) = \sum_f \delta(E - E_f^X)$  and  $\rho_{XX}(E) = \sum_{f'} \delta(E - E_{f'}^{XX})$ , respectively. The mono-exciton and bi-exciton energies are evaluated here without interparticle Coulomb and exchange interactions, that is,  $E(X_{v,c}) = \epsilon_c - \epsilon_v$ , and  $E(XX_{v,c,v',c'}) = \epsilon_c - \epsilon_v + \epsilon_{c'} - \epsilon_{v'}$ , where  $\epsilon_v(\epsilon_c)$  are the valence (conduction) single-particle energy levels. This is a good approximation in PbSe, where the large dielectric constant ( $\epsilon_\infty = 23$ ) effectively screens interparticle interactions. The X and XX DOS's threshold (defined as the energies below which the DOS vanishes) are  $E_g$  and  $2E_g$ , respectively. Interestingly, we find that although



**Figure 2.** (a) Calculated single-particle density of states of a PbSe quantum dot 3.1 nm in diameter. The energy zero corresponds to the valence-band maximum. (b) shows the calculated single-exciton (X) and bi-exciton (XX) densities of states of the same quantum dot, as a function of the renormalized energy  $E/E_g$ , where  $E_g$  is the single-particle band gap of the quantum dot (1.6 eV). The DOS's have been broadened by a 50 meV Gaussian. The fraction of transition matrix elements  $\langle X|W|XX \rangle$  that vanish due to the Coulomb selection rule is shown in part (c) as a function of the renormalized energy  $E/E_g$ .

the X DOS scales approximately linearly with energy in the interval  $2E_g < E < 3E_g$ , the XX DOS scales as  $\rho_{XX}(E) \propto (E - 2E_g)^5$ . The reason is that the number of ways of distributing the energy between two excitons increases much more rapidly, as a function of energy, than the number of single exciton states. As a result, the X and XX DOS's intersect at some energy, which in the case of a 3.1 nm quantum dot is  $2.35 E_g$  (Figure 2b).

Figure 2b reveals the existence of very different densities of final states for the II and AR processes of eqs 1 and 2, suggesting the possibility of different transition rates. However, the density of final states is not by itself a reliable indicator of the actual II and AR transition rates, because some transition matrix elements in eqs 1 and 2 may vanish as a consequence of the selection rules enforced by the two-particle character of the Coulomb interaction. For example, to first order the II process can occur only if one of the two electrons (or one of the two holes) in the final XX configuration occupies the same single-particle level as the electron (or the hole) in the initial X configuration (see Figure 1). Figure 2c shows that indeed a large fraction (>98%) of the transition matrix elements  $\langle X|W|XX\rangle$  vanish as a result of this many-body selection rule. Thus, it is important to take into account the existence of many-body forbidden transitions to obtain accurate estimates of the II and AR rates.

We have calculated the II and AR rates by identifying the transition matrix elements that are zero (many-body forbidden), and assuming that the non-zero matrix elements are approximately constant. We have averaged the II and AR rates over the initial states in a narrow energy window (20 meV), to account for (i) the electronic dephasing of the initial state, which we assume to occur on a very fast time scale, and (ii) the statistical nature of ensemble measurements. The ensuing transition rates are shown in Figure 3. Remarkably, we find that the AR rate ( $XX \rightarrow X$ ) is almost constant as a function of energy. This is a consequence of the fact that the nearly linear increase of the density of final states (X) with energy (Figure 2b) is compensated by the increasing fraction of transition matrix elements that vanish (Figure 2c). The II rate ( $X \rightarrow XX$ ), however, increases rapidly with energy, overtaking the AR rate already at  $\sim 2.1E_g$  (Figure 3). For  $\hbar\omega = 3E_g$ , we find that the II rate is over 2 orders of magnitude faster than the AR rate, thus explaining the observed<sup>5,6</sup> asymmetry between AR and II lifetimes.

The II lifetime can be estimated from the calculated ratio between the II and AR rates (Figure 3). The experimental

AR lifetime (extrapolated to the size of the PbSe dot considered here) is approximately 6 ps,<sup>5</sup> which gives, from our calculations, an estimated II lifetime  $\tau^{II} \approx 0.01$  ps at  $\hbar\omega$

$$= 3E_g$$

from Figure 4, the XX-producing absorption spectrum has a well-defined energy threshold at  $\hbar\omega_{\text{th}} \approx 2.2E_g$ , indicating that carrier multiplication can be initiated by photons of this energy. We note, however, that for  $\hbar\omega = 2.2E_g$  only about  $1/4$  of the absorbed photons are potentially useful for II (ratio between the XX-producing absorption spectrum and the full absorption spectrum in Figure 4). This reduces the carrier-multiplication efficiency compared to  $\hbar\omega = 3E_g$ . Our calculated carrier-multiplication threshold is in agreement with the experimental observations of Ellingson et al.,<sup>6</sup> without the need, however, to invoke optically forbidden transitions. We find that the low-energy threshold is due instead to the asymmetry between the valence-band and conduction-band densities of states (Figure 2a), which allow optical transitions where the excess energy of the electron is significantly larger than that of the hole.

Our calculations show (Figure 3) that, for energies above  $\sim 2.2E_g$ , the intrinsic II rate ( $X \rightarrow XX$ ) is much faster than the AR rate ( $XX \rightarrow X$ ). However, the II decay channel competes with other radiative (e.g., electron–hole recombination) and nonradiative (e.g., electron/hole phonon-assisted cooling) decay paths for the photogenerated exciton. Radiative recombination times in PbSe quantum dots are rather long ( $> 1$  ns). Nonradiative intraband relaxation times ranging between 0.25 and 6 ps have been reported in the literature.<sup>19,20</sup> For exciton energies just above threshold ( $\hbar\omega \approx 2E_g$ ), intraband relaxation is faster than II, leading to relatively small<sup>6</sup> or negligible<sup>7</sup> carrier multiplication efficiencies. For  $\hbar\omega \approx 3E_g$ , however, II is significantly faster than any competing decay processes and represents the main decay channel for an electron–hole pair of that energy (Figure 1). For even higher excitation energies, multiple carrier generation can occur through a chain process, whereby the bi-exciton generated by the II process decays into a tri-exciton, and so on. The physical reason for the ultrafast II decay rate lies in the very sharp increase of the multi-exciton DOS with energy (see Figure 2b). This is likely to be a common feature of many semiconductor quantum dots, suggesting that ultrafast II rates can be expected in other materials as well. Indeed, efficient carrier multiplication has been reported recently in CdSe,<sup>7,8</sup> PbS,<sup>6</sup> and PbTe<sup>21</sup> quantum dots.

In summary, we have proposed that the impact ionization mechanism can explain carrier-multiplication phenomena

recently observed in PbSe colloidal nanocrystals and that no alternative models are required to explain existing experimental observations of (i) ultrafast carrier-multiplication rates ( $< 200$  fs)<sup>5,6</sup> and (ii) low-energy carrier-multiplication thresholds ( $\hbar\omega < 3E_g$ ).<sup>6</sup> In the case of 3.1-nm-diameter PbSe quantum dots, our pseudopotential calculations predict a II threshold  $\hbar\omega_{\text{th}} \approx 2.2E_g$  and a II lifetime smaller than 20 fs for  $\hbar\omega \geq 3E_g$ .

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