strained variations in appendix C.

is stationary for all variations of $(x_0, x_1, \ldots, x_n)$ with x_0 and x_n held fixed. This yields the Euler–Lagrange equations

2.1. Hamiltonian formulation

[redacted]

in the Lagrangian description before translating them into the Hamiltonian represen-

[redacted]

parity-reversal (ℙ) symmetries of a dynami-

[redacted]

cal system as the properties that, given any orbit segment $(x_0, x_1, \ldots, x_n)$, the sequences $(x_n, x_{n-1}, \ldots, x_0)$ and $(-x_0, -x_1, \ldots, -x_n)$, respectively, are also orbit segments. In terms of

[redacted]

constant term in eq. (2.10) or eq. (2.11) analogous to K in eq. (2.7).

[redacted]

the condition that

[redacted]

$$F(x, x^*) = F(-x^*, -x) + R(x) - R(x^*), \tag{2.12}$$

[redacted]

2.6. Examples

As an example, consider the generalized standard map

[illegible]

This satisfies eq. (2.10) with $Q(x) = -V(x)$,

where k is the nonlinearity parameter. This is an even function so the map

$$x^* = x + y - \frac{k}{2\pi} \sin 2\pi x,$$

[illegible]

Theorem 1. True intersections of φ_2-extremizing rotational curves C and C^* generated by an invertible circle map ρ belong to families which are orbits under the area-preserving map T.

To see this, let there be a true intersection at $\theta = \theta_0$. That is, let $\Delta Y(\theta_0) = 0$. Then the

(6.7) and $x_n \equiv x_0 + m$. Then the first variation of the action

$$W_{m,n} \equiv \sum_{j=0}^{n-1} F(x_j, x_{j+1}) \tag{6.8}$$

is zero because $\Delta Y(\theta_j)$ is zero. Calculating the

odd part of $\Delta Y^{>}(x)$ to the cubic correction in

eq. (9.16)

$\Delta Y^{>}(x)$ ... $a\,x^2$

segments which we seek lie on the unstable man-

ifold of this fixed point. (The anticausal solu-

We can now calculate $\Delta Y^{>}(x) = -(k/2\pi) \times \sin 2\pi b(x)$, find its cubic component, and

(the correspondence between orbits and intersections is not complete when the associated cirrotation we have full control over their rotation numbers. It is these solutions which would ap- [redacted] ized action–angle representation. One could use a truncated Farey tree construction to define the principal resonances in the domain of interest [redacted] tween each resonance) and use the curves C, C^*, or the time-symmetric curve specified parametrically by $x = X(\theta)$, $y = \frac{1}{2}[Y_+(\theta) + Y_-(\theta)]$ to define a basic ladder of new momentum co- [redacted] ciently small k. The transformation to the new phase-space coordinates would then be com- [redacted] ing to all irrational rotation numbers since these are not in general smooth and are not continuously connected to the resonance surfaces).

We have studied only the lowest order reso- [redacted] a rotational invariant curve or a cantorus. In the former case φ_2 is obviously a local (and global) [redacted] tions as the control parameter is varied would also be interesting to investigate, as well as the implications of this method for the theory of transport in area preserving maps.

[redacted]

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Appendix A. Circle map identity

van-In

[redacted] used to prove relationships (sum rules) between the Fourier coefficients of a circle map, its sum-difference representation and its inverse. We shall work in x-space, though similar relations [redacted]

$$\int^{1} F(x^* - x)\,[x'_+(\eta) - x'_-(\eta)]\,\mathrm{d}\eta \equiv 0, \quad (A.1)$$

[redacted]

for *any* integrable function $F(x) = f'(x)$. Here $x^* - x$ is a shorthand for $x_+(\eta) - x_-(\eta)$. Equation (A.1) follows by recognizing that the integrand is the perfect differential $\mathrm{d}f(x^* - x)$ and [redacted]

$$\int^{1} F(x^* - x)\,x'_-(\eta)\,\mathrm{d}\eta$$

[redacted]

$$\equiv \frac{1}{2}\int_0^1 F(x^* - x)[x'_+(\eta) + x'_-(\eta)]. \quad (A.2)$$

In particular, choosing $F(\cdot) \equiv \cdot$ and $\eta =$ [redacted] simply $-\Omega$.

Appendix B. Time-symmetric representation

[redacted] otherwise) of the map $\rho : \theta \mapsto \theta^*$ is manifest is