ter the field somewhat, but for moderate plasma pressures the magnetic field lines will tend to remain close to nested tori. Because the magnetic field causes plasma properties to be highly anisotropic, it is desirable to use a curvilinear coordinate system, with level surfaces of one of the coordinates corresponding to the approximately interior functions for the magnetic field systemtype for Hamiltonian systems more susceptible to numerical experimentation than continuous time systems, and are consequently already well explored, especially the standard map (see e.g. MacKay et al. [4]). Since these maps are much simpler than full Hamiltonian systems (especially those corresponding to magnetic field lines in plasma confinement problems) their



strained variations in appendix C.

is stationary for all variations of  $(x_0, x_1, \ldots, x_n)$ with  $x_0$  and  $x_n$  held fixed. This yields the Euler-Lagrange equations

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cal system as the properties that, given any orbit segment  $(x_0, x_1, \ldots, x_n)$ , the sequences  $(x_n, x_{n-1}, \ldots, x_0)$  and  $(-x_0, -x_1, \ldots, -x_n)$ , respectively, are also orbit segments. In terms of the properties the properties bet

$$F(x, x^*) = F(-x^*, -x) + R(x) - R(x^*),$$
(2.12)

## 2.6. Examples

where k is the nonlinearity parameter. This is an even function so the map

	As an example, c dard map	onsider the generalized	stan- <i>x</i> * =	$= x + y - \frac{k}{2\pi} \sin 2\pi$	х,	
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	This satisfies eq. (	2.10) with $Q(x) = -V$	V(x),		ansible. For the st	andoud
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Theorem 1. True intersections of  $\varphi_2$ -extremizing rotational curves C and C<sup>\*</sup> generated by an invertible circle map  $\rho$  belong to families which are orbits under the area-preserving map T. (6.7) and  $x_n \equiv x_0 + m$ . Then the first variation of the action

$$W_{m,n} \equiv \sum_{j=0}^{n-1} F(x_j, x_{j+1})$$
(6.8)

To see this, let there be a true intersection at  $\theta = \theta_0$ . That is, let  $\Delta Y(\theta_0) = 0$ . Then the

is zero because  $\Delta Y(\theta_i)$  is zero. Calculating the

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	We can now calculate $\Delta Y^{>}(x) = -(k/2\pi) \times$	(the correspondence between orbits and inter-	
	$\sin 2\pi b(x)$ , find its cubic component, and	sections is not complete when the associated cir-	
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	rotation we have full control over their rotation numbers. It is these solutions which would ap-	port of the US National Science Foundation, un- der grant NSF-DMS9001103.	
	ized action-angle representation. One could use a truncated Farey tree construction to define the principal resonances in the domain of interest	Appendix A. Circle map identity van-In	
	tween each resonance) and use the curves $C, C^*$ , or the time-symmetric curve specified paramet- rically by $x = X(\theta), y = \frac{1}{2}[Y_+(\theta) + Y(\theta)]$ to define a basic ladder of new momentum co- ordinate surfaces. These survey are assumed not	used to prove relationships (sum rules) between the Fourier coefficients of a circle map, its sum- difference representation and its inverse. We shall work in x-space, though similar relations could coucle map be derived in the $0$ space	
	ciently small $k$ . The transformation to the new phase-space coordinates would then be com- plated by interpolation (rother than by using the	$\int_{-\infty}^{1} F(x^* - x) \left[ x'_{+}(\eta) - x'_{-}(\eta) \right] d\eta \equiv 0,  (A.1)$	
	ing to all irrational rotation numbers since these are not in general smooth and are not continu- ously connected to the resonance surfaces). We have studied only the lowest order reso- nances in detail. It would be interesting to study	for any integrable function $F(x) = f'(x)$ . Here $x^* - x$ is a shorthand for $x_+(\eta) - x(\eta)$ . Equation (A.1) follows by recognizing that the integrand is the perfect differential $df(x^* - x)$ and	
	a rotational invariant curve or a cantorus. In the former case $\varphi_2$ is obviously a local (and global)	$\int_{0}^{1} F(x^{*} - x) x'(n) dn$	
	that a cleara-land minimum an a sentemu.		
	tions as the control parameter is varied would also be interesting to investigate, as well as the implications of this method for the theory of transport in area preserving maps.	$\equiv \frac{1}{2} \int_{0}^{1} F(x^* - x) [x'_{+}(\eta) + x'_{-}(\eta)].  (A.2)$	
		In particular, choosing $F(\cdot) \equiv \cdot$ and $\eta = \frac{1}{2} \int \frac$	
••• ••	One of us (R.L.D.) would like to acknowledge L. Chim for assistance with the analytic single mode calculation and W.A. Coppel and B.G.	sentation of $\alpha^{-1}$ corresponding to eq. (3.2) is simply $-\Omega$ .	Ī
	Kenny for suggesting useful references. It is a pleasure also to acknowledge useful comments from LM Greene and R S MacKay on the re-	Appendix B. Time-symmetric representation	
	tions. One of us (J.D.M.) acknowledges the sup-	otherwise) of the map $\rho: \theta \mapsto \theta^*$ is manifest is	