

This exam is worth 100 points and has 6 problems.

**Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.

**Please begin each problem on a new page.**

**DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.

5. [2360/111622 (20 pts)] Use the Method of Undetermined Coefficients to find the general solution of  $\frac{d^2y}{dt^2} - \frac{dy}{dt} = 40 \cos^2 t$ . Hint:  $2 \cos^2 t = 1 + \cos 2t$ .

6. [2360/111622 (20 pts)] Use Laplace Transforms to solve the initial value problem  $y'' + 2y' = 12e^{-2t}$ ;  $y(0) = 5$ ;  $y'(0) = 0$ . Hint: The following may be handy:  $\frac{1}{s(s+2)^2} = \frac{1}{4} \left[ \frac{1}{s} - \frac{1}{s+2} - \frac{2}{(s+2)^2} \right]$

**Short table of Laplace Transforms:**  $L\{f(t)g(t)\} = F(s) \int_0^1 e^{-st} f(t) dt$

In this table,  $a, b, c$  are real numbers with  $c \neq 0$ , and  $n = 0; 1; 2; 3; \dots$

$$L\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}} \quad L\{e^{at} \cos bt\} = \frac{s-a}{(s-a)^2 + b^2} \quad L\{e^{at} \sin bt\} = \frac{b}{(s-a)^2 + b^2}$$

$$L\{t^n f(t)g(t)\} = (-1)^n \frac{d^n F(s)}{ds^n} \quad L\{e^{at} f(t)\} = F(s-a) \quad L\{f(t-c)g(t)\} = e^{-cs} F(s)$$

$$L\{t^n f^{(n)}(t)g(t)\} = F(s) \frac{d^n F(s)}{ds^n} \quad L\{f(t-c)g(t)\} = e^{-cs} F(s) \quad L\{f(t)g(t-c)\} = e^{-cs} L\{f(t+c)g(t)\}$$

$$L\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - s f^{(n-1)}(0)$$