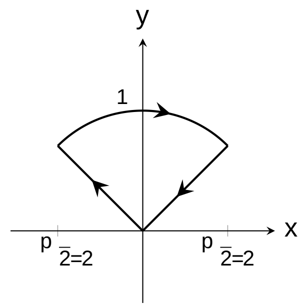
1. [2350/050823 (46 pts)] A wire is in the shape of the curve C given by $\mathbf{r}(t) = t\,\mathbf{i} + t\,\mathbf{j} + t^2\,\mathbf{k};\quad 1 \quad t \quad 2$.

   (a) [9 pts] Does the wire intersect the plane that contains the points $(1; 2; 0); (0; 0; 3); (0; \quad 1; 4)$? If so, nd the point of intersection. If not, explain why not.

   (b) [5 pts] What is the curvature of the wire when $t = 0$?

   (c) [15 pts]

2. [2350/050823 (20 pts)] Consider the oriented curve C, shown in the figure (the curved portion is an arc of the unit circle). Compute the circulation of $\mathbf{V}$ on C where $\mathbf{V} = \left(16y + \sin x^2\right)\mathbf{i} + \left(4e^y + 3x^2\right)\mathbf{j}$.



SOLUTION :

4. [2350/050823 (20 pts)] Compute $\displaystyle\int_C P\,dx + Q\,dy + R\,dz$ where $\mathbf{F} = \langle P; Q; R\rangle = y\,\mathbf{i} + x^2\,\mathbf{j} + z\,\mathbf{k}$ by evaluating an appropriate surface integral. C is the boundary of the portion of the plane $x + y + 5z = 1$ in the first octant, oriented counterclockwise when viewed from above.

S OLUTION :

We use Stokes' Theorem.

$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & x^2 & z \end{vmatrix} = (2x - 1)\,\mathbf{k}$$

The surface is $g(x; y; z) = x + y + 5z$ which we project onto the xy-plane giving R as the triangle with vertices $(0; 0); (1; 0); (0; 1)$ and 1) (0; 0) and Tf 112.52 -5.978 Td [( )]TJ 0e6 Tf 31/F11 20.47.52 -5.978 Td [( )]TJ 0e6 Tf 31/F11 20normalF